Modelling the peristaltic flow of a Bingham fluid

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We present a mathematical model that describes the peristaltic flow of a Bingham fluid in lubrication approximation. A Bingham fluid is a particular type of visco-plastic material that behaves as a viscous linear fluid when the applied stress is larger than a fixed threshold and as a rigid undeformable medium when the stress is below the threshold. Peristaltic flows are generated by the continuous periodic contraction and expansion of the flexible walls. These flows occur in various branches of biomechanics and are of prime importance when considering, for instance, how physiological fluids such as blood and urine are transported in the human body.

To model the system we follow a novel approach developed by our reaserch group in Florence in which the rigid plug momentum equation is derived using an integral formulation for the balance of linear momentum. In practice we model the unyielded domain as an evolving non-material volume. The mathematical problem is formulated for the yielded and unyielded parts and appropriate boundary conditions are established at the walls and at the yield surface. The zero order approximation leads to a system formed by an integral equation and an algebraic equation for the yield surface and for the plug velocity (which is uniform in space) respectively. Because of the integral approach adopted in the unyielded part of the flow, the leading order approximation does not give rise to any inconsistency or lubrication paradox. The problem is solved numerically and an analytical solution is found when the oscillating wall is given as a small perturbation of the uniform wall.

References

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