

“Recent advances in the mathematical modelling of the Fåhræus-Lindqvist effect”

The Fåhræus-Lindqvist (FL) effect is a phenomenon that occurs in blood vessels with diameter in the range $\approx 30\text{-}300\ \mu\text{m}$ and is named after the two Swedish scientists Robin Fåhræus and Johann Torsten Lindqvist[1]. It consists in a progressive reduction of the apparent blood viscosity as the blood vessel radius decreases.

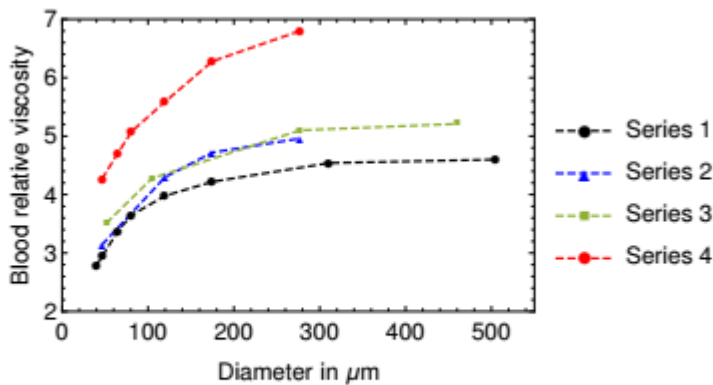


Figure 1: Original data of the Fåhræus-Lindqvist experiment.

The FL effect is clearly related to the rheological properties of blood. Indeed, despite blood is a suspension of various cells (mostly RBCs or erythrocytes), in large tubes like veins and arteries (in the range of millimeters to centimeters) and at relatively high shear rate ($\geq 100\ \text{s}^{-1}$), this fluid shows the characteristic Newtonian behaviour of an incompressible liquid. At a smaller scale this is no longer true and inhomogeneity effects become highly significant and must be considered. Despite the great importance of this topic in physiology, until the fifties, papers on the rheology of blood were scarce and not properly connected in textbooks or manuals dealing with blood or the blood circulation. About seventy years ago (twenty years after Fåhræus-Lindqvist) Copley [2] observed that *Reviews on the viscosity of blood deal largely with data on apparent viscosities. The relative paucity of rheological treatments of blood is contrasted by the large number of observations of rheological phenomena of this humor.*

The Hayne's conjecture [3] of the existence of a “marginal free-cell layer” which reduces the dissipation, addressed many researches in the right direction in the last sixty years, but remained just a qualitative explanation.

Only a few years ago Secomb [4] emphasized the point of major concern:

During the past 20 years, equations for the apparent viscosity of blood in vitro and in vivo have been used extensively in theoretical analyses of blood flow in networks of microvessel. However, these equations are empirical and not derived from analyses of the fluid mechanical phenomena involved. The development, from first principles based on knowledge of the mechanical properties of RBCs and other components of the system, of theories capable of predicting the behaviors described by these equations has proved to be a formidable challenge.

Despite all the efforts and hundreds of studies devoted to the FL effect in the last ninety years, a rigorous explanation based on the first principles of fluid dynamics has been achieved only very recently [5-7]. In this presentation we wish to present and discuss these recent advances in the mathematical modelling of the FL effect.

References

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