Theory of conjectural variations and its application to natural resource management.

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Abstract

The notion of conjectures has a long history in the Theory of Games, since the introduction of conjectural variation equilibria by Bowley (1924), Frisch (1933) and others. According to this behavioral model, a player conjectures a reaction of the rivals in response to a modification of his own actions. This amounts to saying that each player believes that other players are somehow "constrained" to play on specific sets of the space of strategies. The notion of conjectural variation equilibrium in a static game has prompted the interest of numerous researchers but has met no success in theoretical economics. Indeed, in a static setting, this concept describes an inconsistent, not fully rational behavior of agents. An attempt to improve this notion was to endogeneize the conjecture mechanism by requiring a certain consistency among these conjectures (Bresnahan, 1981 among other proposals), unsuccessfully; see for instance Lindh (1992). On the other hand, conjectural variations models have found more success in empirical economics, in the sense that they sometimes match the observed behaviors of firms better than does the standard Nash equilibrium; see Slade (1995). It is "as if" firms had conjectures about other firms, despite the lack of complete rationality of this behavior. The literature on static conjectural variations does not explain why, most likely for lack of a dynamic aspect.
Several papers are devoted to conjectural variations in a dynamic setting. Some of them show that the steady state of the Nash-feedback equilibria of a certain dynamic game is the conjectural variations equilibrium of the associated static game.
Specifically and important for practical situations, in this kind of "procedure", decision makers do not have (do not need) information about their opponents' benefit function and operate based on simple beliefs about their opponents' behaviours. Conjectures replace the need of complete information when computing optimal behaviors.
The idea of this lecture is to present conjectural equilibrium in some dynamic models and compare them with usual behaviors in complete information: the Nash equilibrium and the cooperative solution (définir avant?).
Basically, as said before, players assume that their own actions have an effect on the actions of others players and in the simple learning procedure, players' beliefs are updated based on observations of the actions of the others players or of the evolution of a dynamics over time. In a infinite horizon models, some kind of consistence can be imposed in order to ensure the rationality of the conjectures.
Other than the presentation of general results, an application to natural resource management will be our showpiece.

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