Self-Similar Solutions for the Heat Equation with a Positive non-Lipschitz Continuous, Semilinear Source Term

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We investigate the existence of self-similar solutions for the parabolic equation $u_t = \Delta u + u^m H(u)$, with $0 \leq m < 1$ and H the Heaviside graph, coupled with the initial datum $u(\boldsymbol{x}, 0) = -c\left(|\boldsymbol{x}|^2\right)^{\frac{1}{1-m}}$, with $c > 0$. We analyze two cases: the problem in $\mathbb{R}^n$, $n > 1$, with $m = 0$ and the problem in $\mathbb{R}$ when $0 \leq m < 1$. In the first case we extend the result of Gianni and Hulshof, "The semilinear heat equation with a Heaviside source term", EJAM, 1992, and show that there exist only two self-similar solutions changing sign, provided $0 < c < c_{cr}$, with c_{cr} obtained solving a specific algebraic equation depending on n. In the second case we prove that there exist at least two self-similar solutions of problem $u_t = u_{xx} + u^m H(u)$, $u(x, 0) = -c\left(x^2\right)^{\frac{1}{1-m}}$, changing sign and evolving region where $u > 0$. These solutions are of great interest. Indeed, on one hand they prove that the problem does not admit uniqueness and on the other they prove that a single point where $u(x, 0) = 0$, for an initial datum which is otherwise negative, can generate a region where $u(x, t)$ is positive.