ON THE WELL-POSEDNESS OF NONLINEAR PROBLEMS

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We start by recalling the concepts of well-posedness for minimizations problems, in the sense of Tykhonov and Levitin-Polyak. We proceed with well-posedness concepts for variational and hemivariational inequalities. These concepts are based on two main ingredients: the existence of a unique solution and the convergence to it of the so-called approximating sequence.

Inspired by these properties, we introduce a new concept of well-posedness for abstract problems in metric spaces, the so-called $\mathcal{T}$-well-posedness concept. The theory of $\mathcal{T}$-well-posedness problems we construct gives necessary and sufficient conditions which guarantee the convergence to the solution of a nonlinear problem, unifies different convergence results and provides a framework in which the link between different problems can been established. It can be used in the study of a large class of nonlinear problems like fixed point problems, minimization problems, inequality problems, inclusions, for instance.

Finally, we illustrate the theory in the study of a class of variational inequalities in reflexive Banach spaces. As an application we consider a mathematical model which describes the equilibrium of an elastic body with an obstacle, the so-called foundation. We apply our results in the study of this contact problem and derive convergence results together with the corresponding mechanical interpretations. We also present numerical simulations which validate these convergence results.